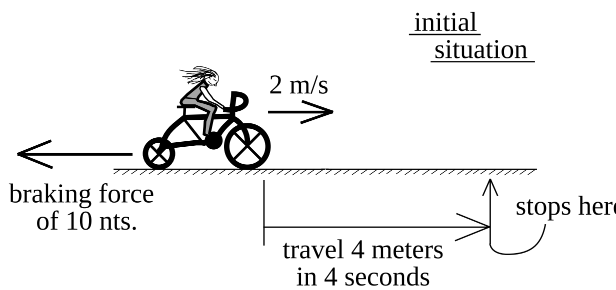
CHAPTER 7 -- MOMENTUM

QUESTION SOLUTIONS

7.1) A net force F stops a car in time t and distance d. If you multiply that force by the time over which it is applied, what will that quantity tell you?

Solution: When a net force is applied to a body over a given time, the product of the force and the time gives you what is called the *impulse* applied to the body. The *impulse*, as a vector, is equal to the amount of *momentum change* the body experiences due to the application of that force over that period. The general relationship, called *the impulse expression*, is $\mathbf{F}_{net}\Delta t = \Delta\mathbf{p}$, where the momentum vector $\mathbf{p}$ is equal to $m\mathbf{v}$. (Just to be clear, the vector called *momentum* is designed to embody the two parameters that must be contended with when one wants to determine just how large an applied force must be to stop a moving body in a given amount of time--those parameters are the body's inertia (its mass) and how fast it is moving (its velocity) . . .)

7.2) Your little sister is riding her trike. She is moving with velocity *2 m/s*. She wants to stop, so she lightly engages the brake pedal. A net force of *10 newtons* is applied to the wheels bringing the trike to a stop in *4 seconds* over a distance of *4 meters*. She decides to experiment (she's a precocious little thing), so:



a.) She doubles her trike speed to *4 m/s* and tries to stop the trike in twice the time (*8 seconds*), distance be damned. How large a force must she apply?

Solution: In fact, we really don't need to use the numbers here at all. All we need to know is that she doubles her speed and doubles her time. The force/velocity relationship that is applicable is the *impulse expression*. It states that $F_1\Delta t = m(\Delta v)$. From that it seems obvious that if, indeed, we double both the *time* and the *velocity change*, the force will remain the same.

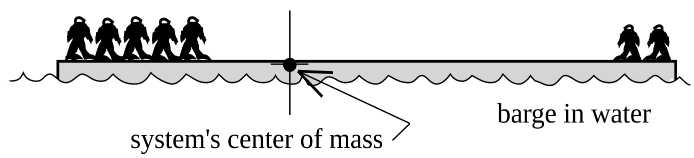
b.) To make things even more exciting, she gets the trike back up to *4 m/s* again and tries to stop it over twice the distance (*8 meters*), time be damned. How large a force must she apply?

Solution: The force/distance expression we need in this case is the *work/energy theorem*. It states that the work done by a force $\mathbf{F}$ over a distance $\mathbf{d}$ is equal to the body's *kinetic energy change*, or $\mathbf{F}\cdot\mathbf{d} = .5m\,\Delta\mathbf{v}^2$. If we double the distance, the left side of the expression will increase by a factor of two. If we double the velocity, the right-hand side will go up by a factor of four. Doubling both, therefore, will require the force to double if the two sides of the equation are to remain equal.

c.) Is there a difference in these two force quantities and, if so, why?

Solution: Yes, there is a difference. The key is most easily seen by starting with the impulse equation. We know that $F\Delta t = \Delta(mv)$, so when we double the initial velocity (i.e., double $\Delta(mv)$) and double Δt , it's pretty obvious that the force will not change. The problem arises when we double both the velocity and the distance over which the force acts. The reason this is a problem is because doubling the distance *doesn't* double the time. Specifically, if the force, hence acceleration, is constant, the relationship between d and t is summarized in the kinematic expression (in its simplest form) $d = .5at^2$. Substituting F/m in for a, this becomes $d = .5(F/m)t^2$. Solving for time yields $t = (2dm/F)^{1/2}$. When we put this into our impulse equation $F\Delta t = \Delta(mv)$, we get $F[(2dm/F)^{1/2}] = \Delta(mv) = 0 - mv$ (remember, the final velocity is zero). Squaring both sides and canceling out the m and F terms yields (lo, and behold) $Fd = .5mv^2$. Note that this relationship is the *work/energy theorem* we used in drawing our conclusions for *Part b* . . . and isn't it interesting how the energy and impulse relationships fit together?

7.3) Five 300 pound football players stand at one end of a relatively light barge (a few thousand pounds) facing two 200 pound football players at the other end. The *center of mass* of the barge/player system is shown in the sketch. Someone blows a whistle and the five 300 pounders run toward the two terrified, stationary 200 pounders, tackling them in a heap at the right end of the barge. Ignoring the frictional effect water would bring to the system (this would probably be considerable, but ignore it anyway):



a.) In which direction has the *center of mass* of the barge/players system shifted as a consequence of the motion of the five monsters?

Solution: What really happens when you run? Your feet push back on the ground (think about it--if you're in loose dirt, in what direction does the dirt fly as you begin to run? . . . it flies backwards because you are applying a force to it in that direction) as the ground pushes forward on you (if you're going to move forward, you'd better have a force on you in that direction--that force comes from your interaction with the ground). So when the behemoths begin to run to the right, they apply a force to the barge to the left. Those two forces are equal and opposite (Newton's Third Law . . . which I will henceforth abbreviate as N.T.L.) and the consequence is that the *center of mass* of the system *goes nowhere*. Put a little differently, all of the forces along the line of motion are *internal to the system* (that is, they are the consequence of the interaction of the pieces of the system). Because there are no *external forces* to change the system's net motion, the motion of the *center of mass* remains the same. It was at rest at the start so it will remain at rest throughout the activity.

b.) How would this have changed if, when the horde was halfway to the other end, the two 200 pound players had jumped off the barge (one off each side--*not* off the end)?

<u>Solution</u>: It's easy to get confused with all of the motion. The key to deciding if the initially stationary *center of mass* changes position is to ask the question, *"Are there any external forces acting on the system in the direction of interest?"* In this case, there aren't any such forces until the 200 pounders hit the water. That is, the force the 200 pounders provide to the boat as they jump will be the same as the force the boat applies to them (ever try to step out of an untied boat and onto a dock--the boat moves *away from the dock* as you step toward the dock . . . it can be quite an exciting experience). These are internal forces. The 300 pounders provide a force to the barge which, once the 200 pounders jump, will accelerate the barge even more than before, but the boat applies a force to the 300 pounders that is equal and opposite. The net effect is that the *center of mass* of the entire system (200 pounders included) will not vary in its motion (i.e., it'll stay put) until the external force provided by the water stops the 200 pounders dead in their tracks. Then, everything changes.

c.) Would *Part b* have changed if the 200 pounders had jumped off the end, not the side?

<u>Solution</u>: Again, as long as they haven't yet hit the water, all the forces acting on the system will be internal and the motion of the system's center of mass doesn't change.

7.4) A car initially sitting still on a road accelerates to velocity v. The change of the car's momentum is mv. The earth's momentum change is (a) zero, (b) less than mv, (c) mv, (d) more than mv.

<u>Solution</u>: What allows the car to accelerate is its friction-based contact with the earth (if it was sitting on a frictionless surface, it wouldn't accelerate at all). That means the earth is applying a force to the car over some period of time, and the car is applying *the same force* (N.T.L.) to the earth over that same period of time. The impulse is the same in both cases, so the change in momentum for each body will be the same. Of course, the mass of the earth is so large that the earth's velocity change will be minuscule. Nevertheless, the earth's *mass* times that *velocity change* will equal the car's mass times its final velocity.

7.5) The mass and velocity of a golf ball, and the mass and velocity of a basketball both multiply out to equal 12 kg·m/s.

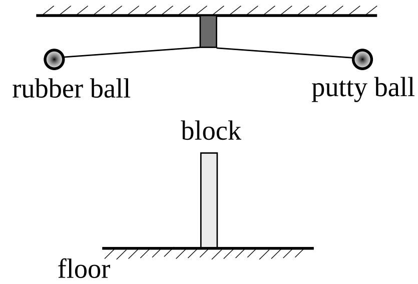
a.) Which will have the larger velocity?

<u>Solution</u>: The product of mv (i.e., the *momentum*) is 12 kg·m/s in both cases. As the golf ball has the smaller mass, it will have the greater velocity.

b.) Assuming you exert the same force, which will take more time to stop?

<u>Solution</u>: The change of momentum is always equal to the impulse $\boldsymbol{F}\Delta t$. As the change of momentum is the same in each case, and as the force is assumed to be the same in each case, the time required to bring each body to rest will be the same. This may seem strange until you remember that in this case, the basketball's *velocity* is small, relatively speaking.

7.6) A rubber ball on a string of length L and a wad of putty on a string of length L have the same mass. If both are pulled to the position shown and released at the same time, both will swing down and strike the wooden block simultaneously. Upon impact, which way, if any, would you expect the block to topple? Explain.

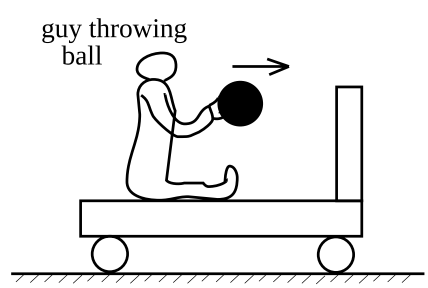


Solution: The block's fall will be governed by whichever mass imparts the greatest force. In fact, that will be the rubber ball. How so? The putty will strike the block and squish, so it will have a final momentum of approximately zero (I'll assume the block tips slowly just after the collision) and its momentum change (Δp) will be approximately $0 - (-mv) = mv$, where v is its *just before collision* speed. On the other hand, the rubber ball will hit and bounce. Having the same *before collision* speed as the putty, its *in* momentum and *out* momentum will have approximately the same magnitude (I'm assuming little energy is lost in the collision) and its momentum change will be approximately $-mv - (+mv) = -2mv$. In other words, the ball's momentum change will be approximately twice that of the putty. According to the impulse equation, a body's momentum change will be equal to the net force applied to the body times the time over which the force acts (i.e., $F\Delta t = \Delta p$). If we assume the *time of collision* for the ball and wad are approximately the same, then the rubber ball's momentum change will be almost twice that of the putty. The block will, therefore, feel approximately twice the force due to the ball than it does due to the putty and, as a consequence, it should tip to the right.

7.7) Assuming both are moving with the same speed, which takes more force to stop, a large truck or a small car? If you said the large truck, you may be wrong. How so?

Solution: A large truck will definitely have greater momentum when compared to a small car traveling with the same speed. The trickiness here is in the fact that stopping a vehicle is not solely a function of force applied, it is also a function of the amount of time the force is applied ($F\Delta t = \Delta p$). If the times of application had been the same for both vehicles, then the force required to stop the truck would be greater. But nothing was said about the time of force application, so nothing can be said about which vehicle will require the greater stopping force.

7.8) A guy finds himself sitting motionless on a cart. He takes a massive medicine ball (this is like a very heavy basketball) sitting on the sled and throws it against the front wall. It collides and bounces back to him. All collisions are elastic (i.e., energy is conserved), and assume the wheels are frictionless.



a.) What is the motion of the cart, if any, after you throw the ball but before it hits the front wall?

Solution: The initial momentum of the system is zero. All the forces acting in the system are internal, so you'd expect the system's *center of mass* to remain stationary.

Does this make sense? Sure it does. When you throw the ball, you apply a force to it that sends it to the right with positive momentum. At the same time, the ball applies a force to you sending you (and the cart) to the left with negative momentum. The total momentum of the system still adds to zero, and even though you, the cart, and the ball are all now in motion, the *center of mass* remains motionless.

b.) What is the motion of the cart after the ball has hit the front wall but before he catches it upon its return?

Solution: There are still no external forces acting, so the system's *center of mass* will remain stationary even though the ball, having hit the front wall, will reverse its direction and move to the left while the cart, having responded to the ball, will also have reversed its direction and end up moving to the right. Note: It might be tempting to suggest that with a lighter ball, the ball's contact with the wall would not provide enough force to reverse the cart's direction. The flaw here is in the fact that if the ball had been considerably lighter, the initial push would have motivated it to a considerably higher speed while its action on the cart and driver would have only minimally accelerated that relatively massive structure. In that situation, in other words, the slow moving cart would have been hit by the light but fast moving ball, and the reversal would still have happened.

c.) What is the motion of the cart after he catches the ball off the rebound?

Solution: As you catch the ball, the force it applies to you stops you and the cart while the force you apply to it stops it. After the catch, everything is stationary. As there have been no external forces acting throughout the motion, the system's *center of mass* will not have changed during any of the interchanges . . . and that's the truth (pthwww).

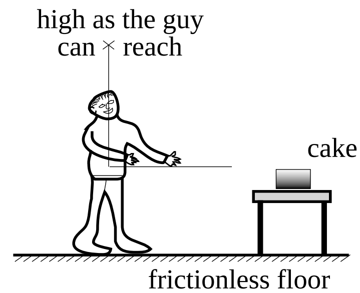
d.) Might this be a way to get the sled's center of mass to move to the right? Explain the usefulness of this approach, if any.

Solution: It would work only if you wanted the sled to move to the right for an instant, and at that it would move to the right only after the bounce and before the catch . . . which is to say, only after having moved to the left first.

7.9) Someone has built a miniature pistol that fires real bullets. Assuming the bullets have the same mass as the pistol, what little problem are you going to run into when you fire the gun?

Solution: Oops. When a bullet is fired, the fixed shell casing in the chamber feels a force that is backward due to the explosion while the bullet feels a force that propels it forward. As the forces are internal to the bullet/gun-casing system, momentum has to be conserved. In other words, under all circumstances, the bullet's momentum will be equal and opposite the momentum imparted to the gun (via the fixed shell casing inside the gun). Under *normal* circumstances, that isn't a problem. The bullet's momentum is made up of a small-mass projectile moving at high velocity while the gun's momentum is comprised of the large-mass gun moving with a small recoil velocity (this is observable as the gun's kick if you happen to be holding it). But in our scenario, the mass of the gun and the mass of the bullet are the same, so the bullet's velocity and the gun's recoil velocity will be the same and the individual firing the weapon will find him or herself holding an object that wants to travel at, maybe, three hundred meters per second. Not surprisingly, this is a *bad* idea.

7.10) You are wearing a beltless pair of pants while standing on a frictionless sheet of ice. There is a table with a huge cake on it that is just out of reach (see sketch--it's to scale). How do you get to the table and the cake?



Solution: The temptation is to believe that you could somehow reach or, if all else fails, fall to the table. The problem is that because there are no external forces available here (the floor is frictionless), the *x coordinate* of your *center of mass* has to stay put. The only way the upper part of your body can reach out toward the table is if the lower part of your body extends out away from the table in the opposite direction (in my country, we call this *falling down*). The problem is that as you rotate around your *center of mass*, your hands will not extend out far enough to reach the table. To get to the table, you must do something that will motivate your *center of mass* to move toward the table. In this case, the easiest thing to do is to take your jacket off and throw it away from the table. Momentum will be conserved, and as there is material (i.e., your jacket) traveling toward the left, there must be material (i.e., your body) moving to the right. The more mass you can jettison (and the faster you can throw it), the quicker you will proceed toward the table.

7.11) A friend inadvertently shoots you with a low powered bee bee gun. In theory, which would hurt more, for the bee bee to hit and stick without penetrating or for the bee bee to hit and bounce? Explain.

Solution: According to the impulse expression, hitting and bouncing would require more force to effect (this assumes that the *time of contact* would be the same for both situations). As a consequence, hitting and bouncing should hurt more.

7.12) Jack (the idiot) fixes a large fan to his sailboat (note: he may be an idiot, but he's a rich idiot) thinking the boat will move forward if he directs the fan toward the sail. Will this work? Explain.

Solution: Let's assume the fan is attached to the boat. To motivate air to move to the right, the fan must provide a force on the air in that direction. In doing so, the air will provide an equal and opposite force back on the fan (and boat), pushing it to the left. When the air hits the sail, it will provide a force to the sail in the direction of the air flow or, in this case, to the right. In theory, assuming that air friction hasn't taken too much of the umph (this is a highly specialized scientific term--an SAT word, in fact--that you will be tested on before graduation) out of the air stream, the force on the sail should be approximately the same as the original air-produced force on the fan (and boat), and the net effect should be nil. In reality, it is probable that the air hitting the sail will not have the same amount of umph as the air force on the fan, and if the boat moves at all, it will move *backwards*.

7.13) What makes a padded dashboard in a car safer than a hard dash (yes, it's the padding, but WHY is that safer)? Relate this to the idea of *impulse*.

Solution: If someone's head collides with a padded dash, the slowing force provided by the dash will occur over a longer period of time than will be the case with a solid dash. The impulse expression states that $F\Delta t = \Delta p$. As the change of momentum is going to be the same in both cases, a longer *time of collision* suggests a lesser force required to effect the momentum change. That translates into a better chance of survival.

7.14) When a ball freefalls, is momentum conserved? Explain.

Solution: Whether momentum is conserved or not in a given situation depends upon what you take to be *the system*. If you take the ball by itself to be the system, then the answer is no (there is an external force--gravity--motivating it to pick up speed, so the momentum of the single object system will change). If you take the ball and earth to be the system, then the answer is yes. The force on the ball and the force on the earth will be an action/reaction pair, and with no external forces acting on the system, the *total* momentum will not change. Of course, *measuring* the earth's momentum change would be right next to impossible, but in theory, momentum would be conserved.

7.15) A railroad engine is attached to a very long train. The engineer wants to move the train forward. Why does he first put the train engine in reverse and push everything backward just a hair before starting forward?

Solution: The attempt to move forward without first backing up would require the train's engine to motivate the entire mass of the train to move from rest all at once. That would take a huge force acting over a large period of time. By backing up, the engineer pushes the couplings between the cars together creating slack between each car. In that way, the only thing the engine has to move initially is itself and the first car. Once that car's inertia is overcome, the engine can deal with the second car's inertia, then the third's, etc. In other words, the engine executes the momentum change in stages.

7.16) Assuming both collide with the same initial speed and direction, and assuming both have the same mass and bounce off with the same speed, which applies a larger force to a wooden block, a rubber ball or a metal ball?

Solution: The impulse expression is $F\Delta t = \Delta p$. We know that the momentum change is the same for both bodies because Δmv is the same in both cases. That means $F\Delta t$ must be the same. Given that product, the ball that feels the greatest force will be the ball that is in contact with the block for the *least amount of time* (i.e. has the smallest Δt). That would *not* be the rubber ball (being rubber, it will give). In short, the steel ball will absorb and impart a greater force.

7.17) Two objects have the same momentum. One is twice as massive as the other. Which requires more work to stop? Explain.

Solution: This is nasty. Momentum is *mv*, so the object with twice the mass must have half the velocity if the momenta are to be the same. In short, we know the ratio of masses (that was given) and velocities (we got that from the equal momentum stipulation). What the *work done* does in this case is to stop each object. That is the same as saying that the work removes enough energy from the system so that each mass's kinetic energy drops to zero. But kinetic energy is a function of mass and velocity *squared* ($KE = .5mv^2$). In other words, if the masses were the same (they aren't, but if they were), an object with twice the velocity of another would have *four times* the kinetic energy. Of course, in this case the object with twice the velocity has *half the mass*, but that still leaves it with *twice* the energy of the other body and twice the work required to stop it.

7.18) Two objects have the same kinetic energy. One is twice as massive as the other. Which will experience the greater momentum change as they come to rest?

Solution: Having the same kinetic energy means that $.5mv^2$ is the same for both objects. If one has half the mass, it must have $(2)^{1/2} = 1.4$ of the velocity of the other for the energies to be the same (i.e., its kinetic energy will be $.5(m/2)(1.4v)^2) = .5mv^2$). That means its momentum will be $(m/2)(1.4v) = .7mv$. As the momentum of the larger mass is mv, it will have the greater momentum.

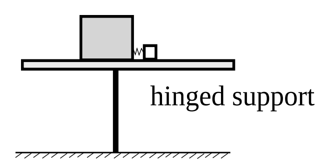
7.19) A system of particles has some non-zero amount of mechanical energy involved within its assembly. Could the system's total momentum be zero? How about the other way around?

Solution: Non-zero mechanical energy means the particles are moving, which means there is velocity involved in the system. But momentum is a vector. If the magnitude of the momentum of the mass moving in one direction equals the magnitude of the momentum of the mass moving in the opposite direction, the two momentum quantities will add to zero. In other words, the energy will be non-zero while the net momentum will be zero. As far as the other way around, a system with momentum is a system in which velocities exist. Kinetic energy is never negative, so there is no way the kinetic energy of one moving object can add to cancel the kinetic energy of another moving object. In short, it isn't possible to have a system with non-zero momentum but zero kinetic energy.

7.20) Responding to a very early paper written by Robert Goddard (he would, with time, become the father of rocket science), a January 1920 editorial printed in the *New York Times* chided Goddard for suggesting that space travel was possible. The article pointed out that without atmosphere to push against, a rocket would go nowhere. Space travel is obviously possible, so how does a rocket go "without atmosphere to push against?"

Solution: According to N.T.L., for every action there must be an equal and opposite "reaction" force (remember, this is a grossly inadequate use of the language--the two forces act simultaneously, not one after the other, but I digress . . .). As the gasses generated by the combustion of the rocket's fuel expand, the rocket effectively applies a force on the gas pushing it out the back while the gas applies an equal and opposite force on the rocket motivating it forward. On a little different tack (i.e., looking at this from the perspective of momentum), all of the forces acting on the rocket/fuel system when in space are internal to the system (remember, there is practically no friction in space). That means that when gas is ejected at high speed from the rear of the rocket, the rocket will move forward with an equal amount of momentum so that the total momentum of the fuel/rocket system is conserved.

7.21) A frictionless beam is attached by a hinge to a thin post. Two blocks of unequal mass have a spring placed between them. They are forced together compressing the spring, then placed on the beam so that the beam balances without tipping.



a.) To begin with, where is the *center of mass* of the system?

Solution: If the beam is in equilibrium, which it evidently is, the *center of mass* must be over a point of support. The only support available is the post, so the *center of mass* must be over the post.

b.) The system is then released with the spring accelerating the blocks outward away from one another. What will the hinged beam do as the blocks move outward (i.e., sit still, rotate, what?)? Use conservation principles to explain your response.

Solution: Being a frictionless system, all the forces acting on either block *along the line of motion* are internal to the system. That means the total momentum of the system will remain conserved and the *center of mass* will continue in whatever motion it had before the uncorking. As it was originally at rest, the *center of mass* will remain over the post and the hinged beam will stay stationary.

7.22) Standing behind a jet engine, you register a force F due to the wind velocity produced by the jet. If the wind velocity doubles, how will the force change?

Solution: Think about what's happening to the air particles hitting you. The force you apply to stop them in time Δt will equal $F\Delta t$. From the impulse expression, that will also equal the change of their net momentum, or $m\,\Delta v$. If the wind velocity doubles to $2v$, the amount of mass that hits you will also double to $2m$ and the new momentum change will become $(2m)(2\,\Delta v)$. In other words, assuming Δt remains the same, the force you must apply to them to bring them to a stop should increase by a factor of four. By N.T.L., the amount of force you apply to the air will be equal and opposite to the force the air applies to you, so the new force you will feel will be $4F$.

7.23) A woman initially standing still on a frictionless ice patch pushes a box that is three times her mass.

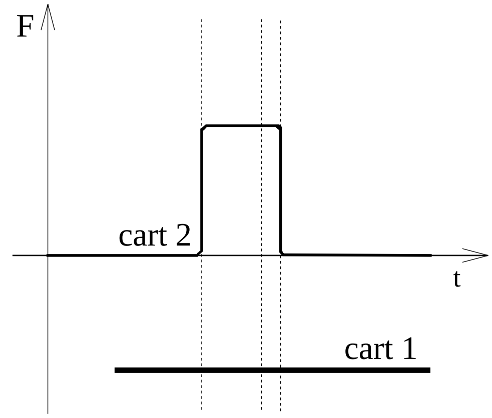
a.) After the push, how will her momentum compare to the box's momentum (i.e., the same, her momentum is less, her momentum is more, what?)?

Solution: The only forces acting within the system are internal (she's pushing on the box, the box is pushing on her), so momentum will be conserved. As the initial momentum in the system was zero (nothing was moving), the magnitude of her momentum and the magnitude of the box's momentum have to be the same after the push. As vectors, each momentum will be opposite in direction.

b.) Explain your response to *Part a* using the idea of *impulse*.

Solution: *Impulse* is related to the force applied and the time over which it is applied. In this case, the impulse on each body is the same (same force due to N.T.L. and same time of application due to their contact with one another). But *impulse* is also equal to the *momentum change* of an object. As both objects started from rest, equal and opposite momentum changes imply that both final momenta will be the same.

c.) For the situation, how will their energies compare? Explain using the idea of work.

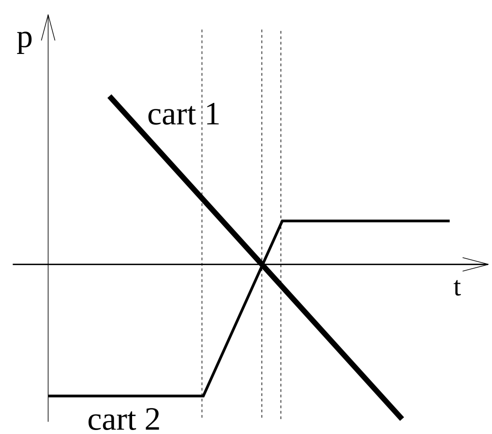


Solution: If the momenta are the same but the masses different, then their velocities will be different. This means their final kinetic energies are different with the more massive (but slower) object having the least kinetic energy. One might think that this would be impossible. After all, it was concluded that the force each experiences is the same. The difference is that the distance over which each force is applied differs. Think about it. The more massive object will move less during *F's* application while the less massive object will move more. The work done on the two will *not* be the same which means the kinetic energy changes will not be the same.

7.24) Which would you prefer to tackle, a 100 kg (220 lb) football player running at 5 m/s or a 50 kg (110 lb) football player running at 10 m/s? Explain.

Solution: Both have the same momentum (500 kg·m/s). The problem is that both don't have the same kinetic energy. The little guy has kinetic energy of $.5(50\ kg)(10\ m/s)^2 = 2500\ newtons$ while the big guy has kinetic energy of $.5(100\ kg)(5\ m/s)^2 = 1250\ newtons$. Clearly the smaller, faster guy is the one to stay away from (in fact, I know this from personal experience having made the mistake of running into just such a fellow during a freshman football game I was playing in when I was in high school--it was a very painful lesson).

7.25) There are two graphs to the right that show the *momentum* versus *time* of two independent carts. What would the *FORCE VERSUS TIME* graph look like for each cart?



Solution: If the area under a *FORCE VERSUS TIME* graph during a differential time interval *dt* is equal to the differential *change of momentum dp* at a given instant (i.e., if *Fdt = dp*), then the slope of a *MOMENTUM VERSUS TIME* graph (i.e., *dp/dt*) will equal the force being applied at a given instant. For *cart 1*, that will be a negative constant. For *cart 2*, that will start out as zero (the momentum isn't changing so the force is expected to be zero), increase suddenly to a constant positive value, then decrease back to zero. Note that the momentum vector changes direction when $\boldsymbol{p} = 0$ (i.e., when the body stops). This has no special significance here. It means only that the force was applied long enough to stop the motion, then get the body moving back in the opposite direction. The *FORCE VERSUS TIME* graphs are shown to the right.

7.26) Would you prefer to be hit by a 8800 pound truck moving at 10 mph, or a marble moving with the same momentum?

Solution: Frankly, I'd prefer neither, but assuming the creator of this problem (oops, I guess that's me) wants an answer, we'll have to think about this. 88 ft/sec (round it off to 90 ft/sec) is approximately equal to 60 mph, so 10 mph is approximately 15 ft/sec. As a meter is approximately a yard, and as 15 ft/sec is 5 yards/sec, the truck's velocity will be approximately 5 m/s. As a 2.2 pound weight is comprised of one kilogram of stuff, our 8800 lb truck has an MKS mass of $(8800\ lbs)/(2.2\ lbs/kg) = 4000\ kg$. That means the truck's momentum is $mv = (4000\ kg)(5\ m/s) = 20{,}000\ kg{\cdot}m/s$. Let's assume the marble's mass is 10 kg (that's a honking big marble, but assume that's what we are dealing with). If its momentum is 20,000 kg·m/s, the magnitude of its velocity must be $v = p/m = (20{,}000\ kg{\cdot}m/s)/(10\ kg) = 2000\ m/s$. Getting hit by a truck moving at 5 m/s isn't going to be a fun experience, but getting nailed by a small, hard, 10 kg ball moving at 2000 m/s is going to do a lot more damage . . . assuming the truck doesn't simply run you completely over. And with that happy, Halloween thought (editorial note: I'm typing this up on the last day in October), I bid you adieu.

7.27) And the last, dying gasp: The momentum of a 1 kg ball moving straight upward is 12 kg·m/s.

a.) What will its momentum be 1 second later?

Solution: Momentum *change* is simply equal to the net external force acting on the system times the time over which the net force acts. In this case, gravity (-mg) is the only external force acting. As such, $F\Delta t$ over a one second interval equals $[m(-g)]\Delta t = (1\ kg)(-9.8\ m/s^2)(1\ sec) = -9.8\ kg{\cdot}m/s$. That is the momentum change over the interval. As the original momentum was $12\ kg{\cdot}m/s$, the new momentum will be the old momentum added to the momentum change over that one second interval will be $12\ kg{\cdot}m/s + (-9.8\ kg{\cdot}m/s) = 2.2\ kg{\cdot}m/s$.

b.) Will the momentum 5 seconds later simply be 5 times the solution to *Part a*? Explain.

Solution: What will be 5 times larger over the 5 second period will not be the final momentum but rather the momentum *change*. That is, $\Delta p_{new} = 5(-9.8\ kg{\cdot}m/s) = -49\ kg{\cdot}m/s$. The new momentum will be the initial momentum added to the momentum change, or $p_{new} = 12\ kg{\cdot}m/s + (-49\ kg{\cdot}m/s) = -37\ kg{\cdot}m/s$.

PROBLEM SOLUTIONS

7.28)

a-i.) The *y component* of momentum before the collision is directed upward while the *y component* of momentum after the collision is directed downward. Clearly, momentum in the *y direction* is not conserved through the collision. This is due to the fact that the force at the ceiling is large enough to change the ball's motion over a minuscule amount of time.

a-ii.) Even if friction was acting in the *x direction* during the collision, the frictional force would be small enough and would be applied over a small enough time to allow the momentum-change during the collision to be negligible. As such, momentum is conserved in the *x direction* through the collision.

b.) As the velocity-magnitude is the same just before and just after the collision, energy was not lost and the collision must have been elastic.

c.) The impulse absorbed by the ceiling as a consequence of the ball's collision with it will be *equal and opposite* to the impulse received by the ball from the ceiling. The ball receives no impulse in the *x direction* (its momentum in that direction is the same before as after the collision) but does receive a change of momentum Δp in the *y direction*. Noting that the ball's initial momentum in the *y direction* (i.e., $p_{1,y}$) is upward (i.e., positive) and its final momentum is downward (i.e., negative), we can write:

$$\begin{aligned}\Delta p_y &= p_{2,y} - p_{1,y} \\ &= (-mv_2 \cos 30^o) - (+mv_2 \cos 30^o) \\ &= -2(\ m \qquad v_2 \quad \cos 30^o) \\ &= -2(.4 \text{ kg})(11 \text{ m/s})\ (.86) \\ &= -7.62 \text{ nt}\cdot\text{sec.}\end{aligned}$$

If the ball's impulse is -7.62 nt·sec, the ceiling's impulse will be +7.62 nt·sec in the *y direction*. This makes sense as the ball's force on the ceiling will be *upward*, hence the *positive sign* on the impulse applied to the ceiling.

d.) The force the ceiling applies to the ball is (-3200 nt)**j**:

$$\begin{aligned}\mathbf{F}\ \Delta t = \Delta \mathbf{p} &= -7.62\ \mathbf{j} \text{ nt}\cdot\text{sec} \\ \Rightarrow \qquad \Delta t &= \Delta \mathbf{p}/\mathbf{F} \\ &= (-7.62\ \mathbf{j} \text{ nt}\cdot\text{sec}) / (-3200\ \mathbf{j} \text{ nt}) \\ &= 2.4\text{x}10^{-3} \text{ sec.}\end{aligned}$$

7.29) Assuming that player #1 is the 60 kg kid and assuming the runners are moving in the *x direction*:

a.) $p_{1,x} = (60\text{ kg})(10\text{ m/s}) = (600\text{ kg}\cdot\text{m/s})$
$p_{2,x} = (120\text{ kg})(5\text{ m/s}) = (600\text{ kg}\cdot\text{m/s})$.

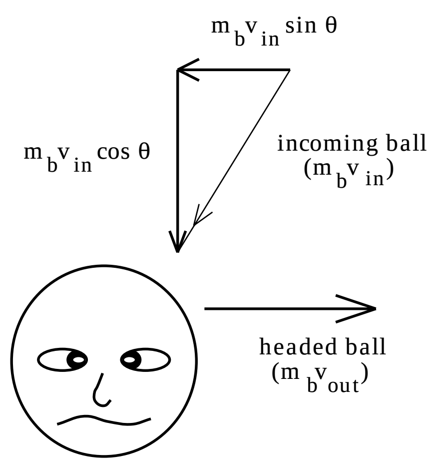
Both players will have the same amount of *momentum*.

b.) $KE_1 = (1/2)m_1{v_1}^2 = .5(60\text{ kg})(10\text{ m/s})^2 = 3000$ joules
$KE_2 = (1/2)m_2{v_2}^2 = .5(120\text{ kg})(5\text{ m/s})^2 = 1500$ joules.

The players have different amounts of energy.

c.) Energy is what can hurt you. Energy is directly proportional to the mass of the moving object, but it is also directly proportional to the SQUARE of the object's velocity. The lesser amount of energy will be imparted by the larger player moving at the slower speed. It should be noted that although it may be more blessed to give than receive, both parties are going to hurt from the collision (Newton's third law--for every action there is an EQUAL and opposite reaction). Both players will feel the same force. The trick, assuming you want to play a sport predicated on the desire to kill someone, is to make the other guy absorb his blow in a more tender place than where you receive yours. That is, your head impacting his knee is not the way to go.

7.30) The sketch shows the incoming and outgoing ball, complete with momentum magnitudes and momentum components.



a.) It is easiest to do problems like this by first writing out what's happening in the *x direction*, then writing out the momentum equation for what's happening in the *y direction*. Subtracting the balls *incoming momentum* from its *outgoing momentum* in both directions yields the *change of momentum* in both directions. Sooo . . .

<u>x direction</u>:

$$\Delta p_x = p_{out,x} - p_{in,x}$$
$$= (mv_2) - (-mv_1 \sin\theta)$$

$= [(.5\ \text{kg})(18\ \text{m/s}) + (.5\ \text{kg})(25\ \text{m/s})(\sin 30^o)]$
$= 15.25\ \text{kg·m/s}.$

<u>y direction:</u>

$$\Delta p_y = p_{out,y} - p_{in,y}$$
$$= (0) - (-mv_1 \cos\theta)$$
$$= (.5\ \text{kg})(25\ \text{m/s})(\cos 30^o)$$
$$= 10.83\ \text{kg·m/s}.$$

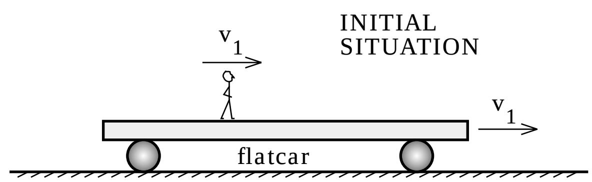
As a vector, $\Delta \mathbf{p} = (15.25\mathbf{i} + 10.83\mathbf{j})$ kg·m/s.

b.) The relationship between *force, change of momentum, and time* is wrapped up in the *impulse equation*. Specifically for the ball:

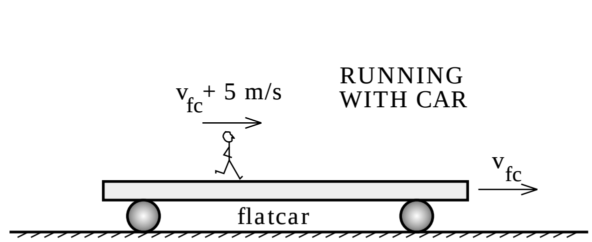
$$\mathbf{F} = \Delta \mathbf{p}/\Delta t$$
$$= (15.25\mathbf{i} + 10.83\mathbf{j}) / (.08\ \text{sec})$$
$$= (190.6\mathbf{i} + 135.4\mathbf{j})\ \text{nts}.$$

This will be equal and opposite the force on your head (N.T.L.).
Note that this is a considerable amount of force. Its magnitude is 270 newtons, or approximately 50 pounds. Also, note that the longer the ball is in contact with the head, the smaller the force is. The moral: from the point of view of your head, it is better to play with an under-inflated ball than an over-inflated ball.

7.31) The bum applies a force to the car; the car applies a force to the bum. As long as the forces are in the direction of the car's motion, all the forces in the direction of motion will be *internal* and momentum will be conserved (note that the velocities are all relative to the stationary track).



a.) Assuming the car is moving in the *x direction*, the bum's mass is m_b, the car's mass is m_c, the initial velocity of both the bum and the car is v_1 in the *+x direction*. After the bum starts running, the final velocity of the car (relative to the ground) is v_{fc} and the final velocity of the bum RELATIVE TO THE GROUND is $v_{fc} + 5\ m/s$:



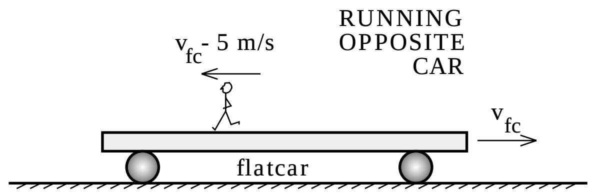
$$\Sigma p_{init} = \Sigma p_{final}$$
$$m_b v_1 + m_c v_1 = m_b(v_{fc} + 5 \text{ m/s}) + m_c v_{fc}$$

$$\Rightarrow \quad v_{fc} = [m_b v_1 + m_c v_1 - m_b(5)]/[m_b + m_c]$$
$$= [(60 \text{ kg})(15 \text{ m/s}) + (800 \text{ kg})(15 \text{ m/s}) - (60 \text{ kg})(5 \text{ m/s})]/[60 \text{ kg} + 800 \text{ kg}]$$
$$= 14.65 \text{ m/s}.$$

(It wasn't requested, but this means that $v_b = v_{fc} + 5\ m/s = 19.65\ m/s$).

Does this make sense? Sure it does. The bum pushes off the car making himself go faster. In doing so, he slows the car just a bit.

b.) With the bum running opposite the direction of the car, the bum's final velocity relative to the car is v_{fc} - *5 m/s*. Following the same steps used in *Part a*:



$$\Sigma p_{init} = \Sigma p_{final}$$
$$m_b v_1 + m_c v_1 = m_b(v_{fc} - 5 \text{ m/s}) + m_c v_{fc}$$

$$\Rightarrow \quad v_{fc} = [m_b v_1 + m_c v_1 + m_b(5)]/[m_b + m_c]$$
$$= [(60 \text{ kg})(15 \text{ m/s}) + (800 \text{ kg})(15 \text{ m/s}) + (60 \text{ kg})(5 \text{ m/s})]/[60 \text{ kg} + 800 \text{ kg}]$$
$$= 15.35 \text{ m/s}.$$

(Again, this means that $v_b = v_{fc} - 5\ m/s = 10.35\ m/s$).

Does this make sense? Again, it does. The bum pushes off the car which makes himself go slower relative to the ground. In doing so, he forces the car ahead.

c.) As the bum runs in a direction perpendicular to the car's motion (say, in the *y direction*), nothing changes in the *x direction*--the car's momentum stays the same. Additionally, because there is an *external force* being provided *by the tracks on the train*, momentum is NOT conserved in the *y direction* as the bum picks up speed.

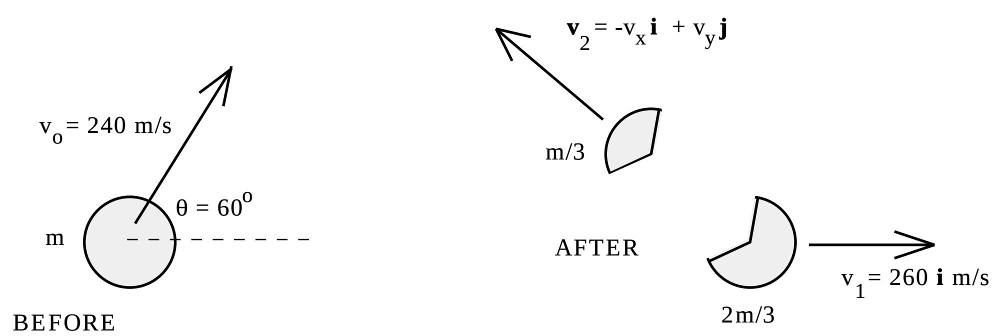
d.) Energy before:

$$KE_{bef} = (1/2)\, m_b v_1^2 + (1/2) m_c v_1^2$$
$$= .5(60 \text{ kg})(15 \text{ m/s})^2 + .5(800 \text{ kg})(15 \text{ m/s})^2$$
$$= 96{,}750 \text{ joules}.$$

$$\begin{aligned} KE_{aft} &= (1/2)\, m_b(v_{fc} + 5)^2 + (1/2)m_c v_{fc}^2 \\ &= .5(60\text{ kg})(19.65\text{ m/s})^2 + .5(800\text{ kg})(14.65\text{ m/s})^2 \\ &= 97{,}433\text{ joules.} \end{aligned}$$

Where did the extra energy come from? The bum did work, burning chemical energy in his muscles as he exerted himself. Some of that energy showed itself as *kinetic energy*.

7.32) A sketch of the situation is shown below:



a.) Although there is gravity acting in the *y direction*, the explosion happens so quickly (i.e., Δt is so small) that momentum will be "conserved through the explosion" in all directions. Writing out momentum considerations in both the *x* and *y directions*, and noting that the signs in v_x and v_y are unembedded, we can write:

--In the *x direction*:

$$\sum p_{before,x} = \sum p_{after,x}$$
$$m(240 \cos 60^o) = [(2/3)m]\,(260) + [(1/3)m](-v_x)$$
$$\Rightarrow \quad v_x = 160\text{ m/s.}$$

Note: This is the *magnitude* of the *x component* of the velocity.

--In the *y direction*:

$$\sum p_{before,y} = \sum p_{after,y}$$
$$m(240 \sin 60^o) = [(1/3)m](v_y)$$
$$\Rightarrow \quad v_y = 623.5\text{ m/s.}$$

As a vector, final velocity of the second piece is, then,

$$\mathbf{v}_2 = (-160\mathbf{i} + 623.5\mathbf{j}) \text{ m/s}.$$

The magnitude of this vector is 643.7 m/s at an angle of 104.4^o.

b.) With *m* equal to 30 kg, the amount of chemical energy converted to kinetic energy is equal to the increase of kinetic energy (i.e., $\sum$KE). This is:

$$\begin{aligned}\Delta KE = \quad & KE_f \quad - \quad KE_o \\ &= [(1/2)(2/3)mv_1{}^2 + (1/2)(1/3)mv_2{}^2] - [(1/2)mv_o{}^2] \\ &= .5[.67(30 \text{ kg})(260 \text{ m/s})^2 + .33(30 \text{ kg})(643.7 \text{ m/s})^2 - (30 \text{ kg})(240 \text{ m/s})^2] \\ &= 1.87\text{x}10^6 \text{ joules}.\end{aligned}$$

This may be an unreasonable figure for a typical explosion, but what do you want from an off-the-wall problem?

7.33) This is a one-dimensional collision problem in which momentum is conserved "through the collision." That means:

$$\begin{aligned}\sum p_{before} &= \sum p_{after} \\ m_8v_8 + m_{10}(0) &= (m_8+m_{10})v_{aft} \\ \Rightarrow \quad v_{aft} &= m_8v_8/(m_8 + m_{10}) \\ &= (880/1880)v_8 \\ \Rightarrow \quad v_{aft} &= .468v_8.\end{aligned}$$

We need a second expression that has v_{aft} in it. We know something about what happens to the energy in the system after the collision, so using the modified conservation of energy approach for the time interval *after the collision up to the complete standstill point*, we get:

$$\begin{aligned}KE_1 \quad\quad & + \sum U_1 + \sum W_{ex} = KE_2 + \sum U_2 \\ (1/2)(m_8 + m_{10})v_{aft}{}^2 & + \quad (0) \quad - f_kd \quad = \quad 0 \quad + \quad 0.\end{aligned}$$

The frictional force f_k is due to m_{10}*'s* brakes locking (m_8*'s* brakes are assumed to remain unlocked). N.S.L. suggests that the normal force on m_{10}

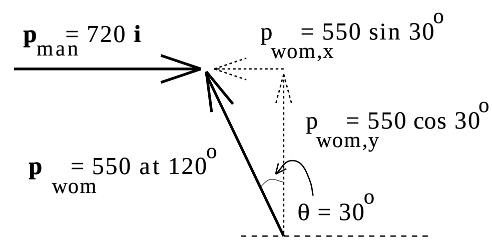
in this case is $m_{10}g$ and that the frictional force is $\mu_k N_{10} = \mu_k m_{10} g$. Substituting this into the above expression and solving for v_{aft} yields:

$$(1/2)(m_8 + m_{10})v_{aft}^2 - f_k d = 0$$
$$\Rightarrow \quad (1/2)(m_8 + m_{10})v_{aft}^2 = \mu_k m_{10} g d$$
$$v_{aft} = [2\mu_k m_{10} g d/(m_8 + m_{10})]^{1/2}.$$

Substituting $v_{aft} = .468v_8$ from above into this expression yields:

$$v_{aft} = [2\mu_k m_{10} g d/(m_8 + m_{10})]^{1/2}$$
$$.468v_8 = [2\mu_k m_{10} g d/(m_8 + m_{10})]^{1/2}$$
$$\Rightarrow \quad v_8 = [2(.6)(1000\ kg)(9.8\ m/s^2)(1.2\ m)/(1880\ kg)]^{1/2}/(.468)$$
$$\Rightarrow \quad v_8 = 5.85\ m/s.$$

7.34) The man's initial momentum is in the *x direction*. The woman's initial momentum is in both the *x* and *y direction*. During the collision, the only forces acting are internal to the *two-person system*. This means momentum will be conserved in both the *x* and *y directions* through the collision. Remembering that the two individuals stick together (it is a *perfectly inelastic collision*), we can approach the problem by looking independently at what has happened to the system's momentum in the *x direction*, then *y direction*. Assuming the final velocity of the two is v_x in the *x direction* and v_y in the *y direction*, we can write:



x direction:

$$\sum p_{before,x} = \sum p_{after,x}$$
$$p_{man\ before,x} + p_{woman\ before,x} = p_{man\ after,x} + p_{woman\ after,x}$$
$$m_m(v_{man\ before,x}) + m_w(v_{woman\ before,x}) = m_m(v_x) + m_w(v_x)$$
$$(90\ kg)(8\ m/s) + (55\ kg)[(-10\ m/s)(\sin 30^o)] = (90\ kg)v_x + (55\ kg)v_x$$
$$\Rightarrow \quad 445 = 145v_x$$
$$\Rightarrow \quad v_x = 3.07\ m/s.$$

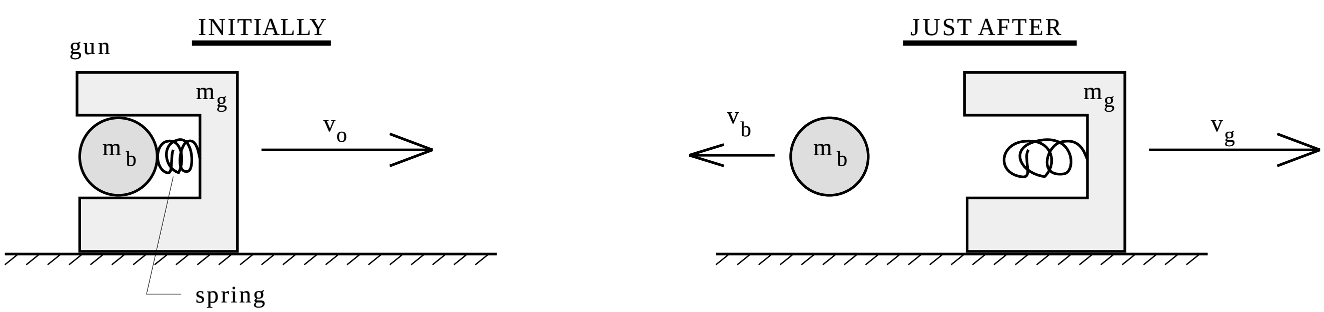
<u>y direction</u>:

$$\sum p_{before,y} = \sum p_{after,y}$$
$$p_{man\ before,y} + p_{woman\ before,y} = p_{man\ after,y} + p_{woman\ after,y}$$
$$m_m(v_{man\ before,x}) + m_w(v_{woman\ before,x}) = m_m(v_y) + m_w(v_y)$$
$$(90\ kg)(0\ m/s) + (55\ kg)[(10\ m/s)(\cos 30^o)] = (90\ kg)v_y + (55\ kg)v_y$$
$$\Rightarrow \quad 476 = 145v_y$$
$$\Rightarrow \quad v_y = 3.28\ m/s.$$

The final velocity of the two as a vector will be:

$$\mathbf{v}_{fin} = (3.07\mathbf{i} + 3.28\mathbf{j})\ m/s.$$

7.35) Momentum is conserved "through the one-dimensional firing of the gun" (see sketch). As such, we can write:



$$\sum p_{before,x} = \sum p_{after,x}$$
$$p_{both} = p_{gun} + p_{ball}$$
$$(m_g + m_b)v_o = m_g v_g - m_b v_b$$
$$(2.04\ kg)(5\ m/s) = (2\ kg)v_g - (.04\ kg)v_b$$
$$\Rightarrow \quad v_g = (.04v_b + 10.2)/2$$
$$= .02v_b + 5.1 \qquad \text{(Equation A).}$$

The spring is ideal so no energy is lost in the gun's firing. Using *conservation of energy* through the firing yields:

$$(1/2)m_g v_o^2 + (1/2)m_b v_o^2 + (1/2)kx^2 = (1/2)m_g v_g^2 + (1/2)m_b v_b^2.$$

Dividing out the *1/2's*:

$$(2\text{ kg})(5\text{ m/s})^2 + (.04\text{ kg})(5\text{ m/s})^2 + (120\text{ nt/m})(.15\text{ m})^2 = (2\text{ kg})v_g^{\,2} + (.04\text{ kg})v_b^{\,2}$$

$$\Rightarrow \quad 53.7 = 2v_g^{\,2} + .04v_b^{\,2}.$$

Substituting *Equation A* in for v_g, we get:

$$53.7 = 2(.02v_b + 5.1)^2 + .04v_b^{\,2}.$$

Expanding yields:

$$.0408v_b^{\,2} + .408v_b - 1.68 = 0.$$

The Quadratic Formula yields:

$$\begin{aligned} v_b &= [-.408 \pm [(-.408)^2 - 4(.0408)(-1.68)]^{1/2}]/[2(.0408)] \\ &= 3.13\text{ m/s or } -13.13\text{ m/s}. \end{aligned}$$

Assuming for the moment that the solution is 3.13 m/s, *Equation A* will give us the gun's velocity:

$$\begin{aligned} v_g &= .02v_b + 5.1 \\ &= .02(3.13) + 5.1 \\ &= 5.16\text{ m/s}. \end{aligned}$$

Assuming for the moment that the solution is -13.13 m/s, *Equation A* will give us the gun's velocity:

$$\begin{aligned} v_g &= .02v_b + 5.1 \\ &= .02(-13.13) + 5.1 \\ &= 4.84\text{ m/s}. \end{aligned}$$

The physical significance of a velocity calculated to be negative, given that we have unembedded the signs on all the velocity terms (hence making them magnitudes), is that the direction of motion has been assumed incorrectly. V_b was assumed to move to the left relative to the ground (i.e., in the negative *x direction*). It is possible we could have been wrong. That is, if the spring had been weak, it would have ejected the ball out the back of the gun, but the ball could have trailed the gun moving slower than the gun but nevertheless to the right in the POSITIVE *x direction*. If that had been the case, we would have computed a negative value for v_b and the negative sign would have told us we had assumed the WRONG DIRECTION for v_b in the first place. That is why we had to at least try the negative velocity value for v_b in *Equation A*.

With v_b negative, the final velocity for the GUN was in the correct direction--to the right--but with LESS VELOCITY MAGNITUDE than it had to start with (it started with 5 m/s velocity--v_g calculates to 4.84 m/s if $v_b = -13.13\ m/s$). Intuition tells us that this is clearly wrong. Conclusion? $V_b = 3.13\ m/s$ making $v_g = 5.16\ m/s$.

7.36)

a.) To stop the cart, Tarzan's momentum must be the same as the cart's but opposite in direction. Momentum will be conserved through the collision. If the system is to be brought to absolute rest (i.e., zero momentum) after the collision, we can write:

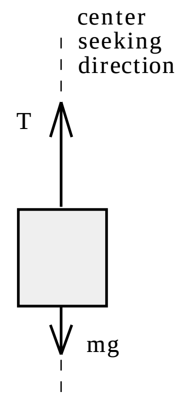
$$
\begin{aligned}
\sum p_{before,x} &= \sum p_{after,x} \\
(p_j + p_c) \quad + \quad p_T &= 0 \\
(m_J + m_c)v_c \quad - m_T v_{bot} &= 0 \\
(40\text{ kg} + 190\text{ kg})\,(11\text{ m/s}) - 90 v_{bot} &= 0 \\
\Rightarrow \quad v_{bot} &= (230\text{ kg})(11\text{ m/s})/(90\text{ kg}) \\
&= 28.1\text{ m/s.}
\end{aligned}
$$

This is the velocity at which Tarzan must move to stop Jane and the cart dead in their tracks.

With this velocity, we can calculate how much energy Tarzan needs at the bottom of his arc. Using *conservation of energy*, we can determine how much of that energy will come from the freefall and how much must come from the run. Using that approach, we write:

$$
\begin{aligned}
KE_{top} \quad + \sum U_{top} \quad + \sum W_{ext} &= KE_{bot} \quad + \sum U_{bot} \\
(1/2)m_T v_{top}^{\ 2} + m_T g h_{top} + (0) &= (1/2) m_T v_{bot}^{\ 2} + (0) \\
\Rightarrow \quad v_{top}^{\ 2} &= v_{bot}^{\ 2} - 2gh_{top} \\
&= (28.1\text{ m/s})^2 - 2(9.8\text{ m/s}^2)(38\text{ m}) \\
\Rightarrow \quad v_{top} &= 6.7\text{ m/s.}
\end{aligned}
$$

b.) The f.b.d. to the right shows tension *up* and weight (i.e., *mg*) *down*. Using N.S.L. to sum the forces in the CENTER-SEEKING DIRECTION, we get:



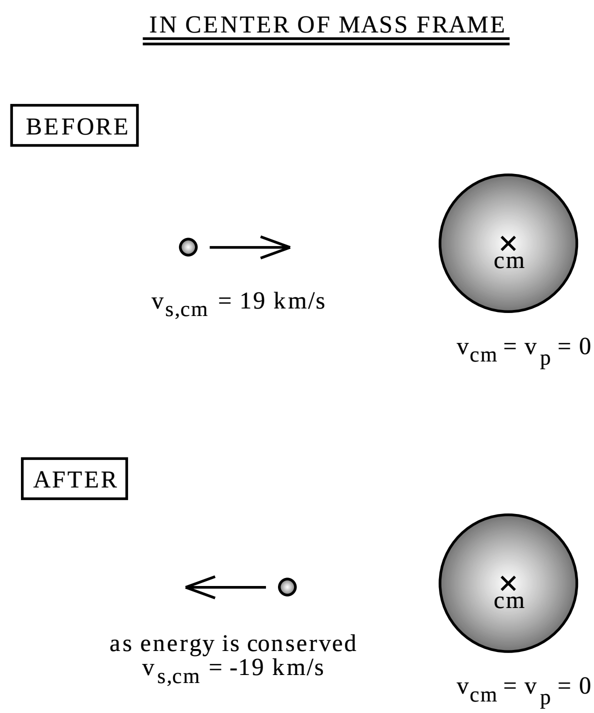
$\underline{\sum F_c}$:

$$T - m_T g = m_T(v^2/r)$$
$$T = m_T g + m_T v_{bot}{}^2/R$$
$$= (90\text{ kg})(9.8\text{ m/s}^2) + (90\text{ kg})(28.1\text{ m/s})^2/(19\text{ m})$$
$$= 4622\text{ nts.}$$

This is over five times Tarzan's weight of 882 newtons.

7.37) Because this is essentially a collision problem, and because the only force acting (gravitational attraction between the two bodies) is internal to the system, momentum will be conserved in this problem. The difficulty lies in the fact that the planet is huge in comparison to the satellite. That is, the momentum of the planet will only change minusculely due to its size. In short, using *conservation of momentum* really won't work here.

There is a clever way to approach the problem, though. Consider it from a *center of mass frame of reference.*



In the free-space frame (i.e., a frame that is stationary relative to both the planet and satellite), the motion of the center of mass and the motion of the planet will, for all intents and purposes, be exactly the same (almost all of the mass in the system is in the planet). That means that in the *center of mass frame*, the planet will appear stationary. It additionally means that the *satellite's incoming velocity* in that frame (i.e., in the center of mass frame) will be $v_{s,cm} = 7\text{ km/s} + 12\text{ km/s} = 19\text{ km/s}$.

If the so-called collision is elastic, energy will be conserved. That means the satellite will leave the collision with the same amount of energy, hence same velocity, as it entered with . . . FROM THE PERSPECTIVE OF THE *CENTER OF MASS FRAME*.

Relative to space (i.e., the lab frame), the velocity of the center of mass and the velocity of the planet are both 12 km/s. That means the velocity of the satellite, relative to space, will be:

$$\begin{aligned} v_s &= v_{cm} + v_{sat.rel.to\ cm} \\ &= 12 \text{ km/s} + 19 \text{ km/s} \\ &= 31 \text{ km/s}. \end{aligned}$$

In other words, the satellite will come into the situation moving with velocity 7 km/s and will leave after interacting with the planet with velocity 31 km/s. This slingshot was used by NASA to boost the speed of both Voyager spacecrafts as they passed by Jupiter.